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Positive Solutions of Several Classes of Boundary Value Problems

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Abstract: This paper is concerned with the existence of positive solutions for second order ordinary differential equations with two-point, three-point and m-point boundary value problems by using an existence theorem. Some sufficient conditions guaranteeing the existence of at least one positive solution of several classes of boundary value problems are established under weaker conditions. The results generalize and improve some previous results.

Keywords: positive solution; existence theorem; boundary value problem

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1 Introduction

It is well known that boundary value problems (BVPs) arise in a variety of different areas of applied mathematics, physics, chemistry and biology, which can be found in the theory of nonlinear diffusion generated by nonlinear sources, in the thermal ignition of gases, in chemical or biological problems, in the vibrations of a guy wire of a uniform cross-section, and in the theory of elastics stability[1-6]. We are interested in the existence of positive solutions for the following second-order differential equations

$$u''(t)+\lambda f\big(t,u(t)\big)=0,\quad t\in(0,1), \tag{1}$$

$$u''(t)+\lambda a(t)f\big(u(t)\big)=0,\quad t\in(0,1), \tag{2}$$

$$u''(t)+f\big(t,u(t)\big)=0,\quad t\in(0,1), \tag{3}$$

$$u''(t)+a(t)f\big(u(t)\big)=0,\quad t\in(0,1), \tag{4}$$

under different boundary conditions

$$u(0)=0,\quad \alpha u(\eta)=u(1), \tag{5}$$

$$u'(0)=0,\quad \alpha u(\eta)=u(1), \tag{6}$$

$$u(0)=u(1)=0, \tag{7}$$

$$u(0)=u'(1)=0, \tag{8}$$

$$u'(0)=u(1)=0, \tag{9}$$

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$$\alpha u(0)-\beta u'(0)=0,\quad \gamma u(1)+\delta u'(1)=0, \tag{10}$$

$$u'(0)=\sum_{i=1}^{m-2} b_i u'(\xi_i),\quad u(1)=\sum_{i=1}^{m-2} a_i u(\xi_i). \tag{11}$$

Such equations arise from the study of radial solutions of partial differential equations on annular regions in R^n, $n\geq 2$, of the forms: $\Delta u+h(|x|)f(u)=0$ and $\Delta u+f(|x|,u)=0$, and contain the generalized Emden-Fowler equations, where $f=u^p$, $p>0$ (gas dynamic, nuclear physics and chemically reacting systems), the Thomas-Fermi equations, where $f=u^{\frac{3}{2}}$ (atomic structures), and so on. In last decades, the existence of positive solutions for the BVPs has been widely studied[7-14]. Inspired by the known results (see, for example, [8-11,13,14]), we attempt to establish a simple criterion for the existence of positive solutions for several classes of BVPs by using an existence theorem in $C[0,1]$ ([15]) and then prove the existence of positive solutions for above BVPs. In this way, our results generalize and improve similar results in [8-11,13,14]. Let us introduce some assumptions for convenience:

(H_1) $\eta\in(0,1)$, $0<\alpha\eta<1$;

(H_1)$'$ $\eta\in(0,1)$, $0<\alpha<1$;

(H_2) $f:[0,1]\times[0,\infty)\to[0,\infty)$ is continuous and $f(t,\cdot)$ does not vanish identically on any subset of $[0,1]$ with positive measure;

(H_2)$'$ $f\in C([0,\infty),[0,\infty))$, $a\in C([0,1],[0,\infty))$ and there is $t_0\in[0,1]$ such that $a(t_0)>0$;

(H_3) α, β, γ, $\delta\geq 0$ and $\rho=\gamma\beta+\alpha\gamma+\alpha\delta>0$;

(H_4) $a\in C((0,1),[0,+\infty))$ satisfies

$$0<\int_0^1 G(s,s)a(s)\mathrm{d}s<\infty,$$

where

$$G(t,s)=\begin{cases}\frac{1}{\rho}(\gamma+\delta-\gamma t)(\beta+\alpha s), & 0\leq s\leq t\leq 1,\\ \frac{1}{\rho}(\beta+\alpha t)(\gamma+\delta-\gamma s), & 0\leq t\leq s\leq 1;\end{cases}$$

(H_5) a_i, $b_i\in[0,\infty)$ satisfy

$$0<\sum_{i=1}^{m-2} a_i<1,\quad \sum_{i=1}^{m-2} b_i<1.$$

We now present the existence theorem which will be used in the latter proofs. Let $E=C[0,1]$ be endowed with the maximum norm

$$\|y\|=\max_{t\in[0,1]}|y(t)|,\quad P=\big\{x\in E\,|\,x(t)\geq 0,\ t\in[0,1]\big\}.$$

Theorem 1[15] Assume $A:P\to P$ is a completely continuous operator and there exist constants b, $c>0$, $0\leq\mu<\nu\leq 1$, $r\in(0,1)$ such that $b<rc$ implies:

(i) $\min\limits_{t\in[\mu,\nu]} Ax(t)\geq r\|Ax\|$ for $x\in P$;

(ii) $\min\limits_{t\in[\mu,\nu]} Ax(t)>b$ for $x\in P$ with $b\leq x(t)\leq\frac{b}{r}$, $t\in[\mu,\nu]$;

(iii) $\|Ax\|\leq c$ for $x\in\bar{P}_c=\{x\in P\,|\,\|x\|\leq c\}$.

Then the equation $Ax=x$ has at least one solution $x^*\in\bar{P}_c$ with $\min\limits_{t\in[\mu,\nu]} x^*(t)>b$.

2 Existence of positive solutions

In this section we will study the existence of positive solutions for aboved-mentioned BVPs. Similar problems have been considered in [7-14]. Particularly, the existence of positive solutions for different BVPs is established in [8-14] by using the Krasnosel'skii fixed point theorem of the cone expansion-compression type. In what follows, we also work in $C[0,1]$ and only the sup-norm is used. For convenience, let $P=\{x\in C[0,1]\,|\,x(t)\geq 0,\ t\in[0,1]\}$ and

$$f_0:=\lim_{u\to 0^+}\frac{f(u)}{u},\quad f_\infty:=\lim_{u\to\infty}\frac{f(u)}{u},\quad \overline{f}_0:=\lim_{u\to 0^+}\max_{t\in[0,1]}\frac{f(t,u)}{u},\quad \underline{f}_\infty:=\lim_{u\to\infty}\min_{t\in[0,1]}\frac{f(t,u)}{u}.$$

Theorem 2 Assume (H_1), (H_2) hold and $\underline{f}_\infty=\infty$. Then, for $\lambda>0$ sufficiently small, (1), (5) has at least one positive solution.

Proof It is easy to see that $u\in P$ is a solution of (1), (5) if and only if

$$\begin{aligned} u(t) &= -\int_0^t (t-s)\lambda f(s,u(s))\mathrm{d}s \\ &\quad -\frac{\alpha t}{1-\alpha\eta}\int_0^\eta(\eta-s)\lambda f(s,u(s))\mathrm{d}s+\frac{t}{1-\alpha\eta}\int_0^1(1-s)\lambda f(s,u(s))\mathrm{d}s. \end{aligned}$$

For $f(t,u)\geq 0$, using the same arguments as those in [7, Lemma 2.2, Lemma 2.3], we can get $u(t)\geq 0,\ t\in[0,1]$ and $\min\limits_{t\in[\eta,1]}u(t)\geq\gamma\|u\|$, where

$$\gamma=\min\left\{\alpha\eta,\ \frac{\alpha(1-\eta)}{1-\alpha\eta},\ \eta\right\}.$$

Define an operator T

$$\begin{aligned} Tu(t) &= -\int_0^t (t-s)\lambda f(s,u(s))\mathrm{d}s \\ &\quad -\frac{\alpha t}{1-\alpha\eta}\int_0^\eta(\eta-s)\lambda f(s,u(s))\mathrm{d}s+\frac{t}{1-\alpha\eta}\int_0^1(1-s)\lambda f(s,u(s))\mathrm{d}s. \end{aligned}$$

It is easy to prove that $T:P\to P$ is completely continuous and $\min\limits_{t\in[\eta,1]}Tu(t)\geq\gamma\|Tu\|,\ \forall\,u\in P$. Thus, the condition (i) of Theorem 1 is satisfied. Now let $\lambda\in(0,\varepsilon)$ be fixed, where $\varepsilon\in(0,1)$ is any given constant. Since $\underline{f}_\infty=\infty$, there is $b>0$ such that $f(t,u)\geq Hu$ for $\forall\,t\in[0,1],\ u\geq b$, where H is chosen so that $\frac{\lambda H\eta(1-\eta)^2\gamma}{2(1-\alpha\eta)}>1$. Thus, for $b\leq u(t)\leq\frac{b}{\gamma},\ t\in[\eta,1]$,

$$\begin{aligned} Tu(\eta) &= -\int_0^\eta(\eta-s)\lambda f(s,u(s))\mathrm{d}s-\frac{\alpha\eta}{1-\alpha\eta}\int_0^\eta(\eta-s)\lambda f(s,u(s))\mathrm{d}s \\ &\quad +\frac{\eta}{1-\alpha\eta}\int_0^1(1-s)\lambda f(s,u(s))\mathrm{d}s \\ &= -\frac{1}{1-\alpha\eta}\int_0^\eta(\eta-s)\lambda f(s,u(s))\mathrm{d}s+\frac{\eta}{1-\alpha\eta}\int_0^1(1-s)\lambda f(s,u(s))\mathrm{d}s \end{aligned}$$

$$
\begin{aligned}
&= -\frac{\eta}{1-\alpha\eta}\int_0^\eta \lambda f(s,u(s))\mathrm{d}s + \frac{1}{1-\alpha\eta}\int_0^\eta s\lambda f(s,u(s))\mathrm{d}s \\
&\quad + \frac{\eta}{1-\alpha\eta}\int_0^1 \lambda f(s,u(s))\mathrm{d}s - \frac{\eta}{1-\alpha\eta}\int_0^1 s\lambda f(s,u(s))\mathrm{d}s \\
&= \frac{\eta}{1-\alpha\eta}\int_\eta^1 \lambda f(s,u(s))\mathrm{d}s + \frac{1}{1-\alpha\eta}\int_0^\eta s\lambda f(s,u(s))\mathrm{d}s - \frac{\eta}{1-\alpha\eta}\int_0^1 s\lambda f(s,u(s))\mathrm{d}s \\
&> \frac{\eta}{1-\alpha\eta}\int_\eta^1 \lambda f(s,u(s))\mathrm{d}s - \frac{\eta}{1-\alpha\eta}\int_\eta^1 s\lambda f(s,u(s))\mathrm{d}s \\
&\geq \frac{\eta}{1-\alpha\eta}\int_\eta^1 \lambda(1-s)Hu(s)\mathrm{d}s \geq \frac{\lambda\eta}{1-\alpha\eta}\int_\eta^1 (1-s)\mathrm{d}s\cdot Hb = \frac{\lambda\eta(1-\eta)^2}{2(1-\alpha\eta)}\cdot Hb > \frac{b}{\gamma}.
\end{aligned}
$$

Consequently

$$
\min_{t\in[\eta,1]} Tu(t) \geq \gamma\|Tu\| > \gamma\cdot\frac{b}{\gamma} = b, \quad b\leq u(t)\leq\frac{b}{\gamma}, \quad t\in[\eta,1].
$$

Therefore, the condition (ii) of Theorem 1 is satisfied. If $q>0$, then

$$
\begin{aligned}
\beta(q) &= \sup_{u\in P,\|u\|\leq q}\ \max_{t\in[0,1]}\Big[-\int_0^t (t-s)f(s,u(s))\mathrm{d}s \\
&\quad -\frac{\alpha t}{1-\alpha\eta}\int_0^\eta(\eta-s)f(s,u(s))\mathrm{d}s + \frac{t}{1-\alpha\eta}\int_0^1(1-s)f(s,u(s))\mathrm{d}s\Big] > 0.
\end{aligned}
$$

For any $c>\frac{b}{\gamma}$, let $\pi=\frac{c}{\beta(c)}$, then for $\lambda\in(0,\varepsilon)\cap(0,\pi]$ and $u\in\bar{P}_c$, we have

$$
\begin{aligned}
Tu(t) &\leq \pi\Big[-\int_0^t(t-s)f(s,u(s))\mathrm{d}s - \frac{\alpha t}{1-\alpha\eta}\int_0^\eta(\eta-s)f(s,u(s))\mathrm{d}s \\
&\quad + \frac{t}{1-\alpha\eta}\int_0^1(1-s)f(s,u(s))\mathrm{d}s\Big] \leq \pi\beta(c) = c.
\end{aligned}
$$

Hence, $\|Tu\|\leq c$, $\forall\ u\in\bar{P}_c$. The condition (iii) of Theorem 1 is satisfied. Thus, for $\lambda\in(0,\varepsilon)\cap(0,\pi]$, all the conditions of Theorem 1 are satisfied. $Tu=u$ has at least one nonzero positive solution. That is, (1), (5) has at least one positive solution.

Theorem 3[15] Assume (H_1), (H_2) hold and there exist b, $c>0$ such that $b<\gamma c$ implies

(i) $f(t,u)\geq lb$ for $\eta\leq t\leq 1$, $b\leq u\leq\frac{b}{\gamma}$;

(ii) $f(t,u)\leq m'c$ for $0\leq t\leq 1$, $0\leq u\leq c$.

Then (3), (5) has at least one positive solution, where γ is given as in Theorem 2 and

$$
l=\frac{2(1-\alpha\eta)}{\gamma\eta(1-\eta)^2}, \quad m'=\frac{2(1-\alpha\eta)}{(2-\alpha\eta+\alpha\eta^2)}.
$$

Using the same arguments as those in Theorem 2, we come to the following conclusions.

Theorem 4[7] Assume (H_1), (H_2)$'$ hold and $f_\infty=\infty$. Then, for $\lambda>0$ sufficiently small, (2), (5) has at least one positive solution.

Theorem 5 Assume (H_1), (H_2)$'$ hold and there exist constants b, $c>0$ such that $b<\gamma c$ implies

(i) $f(u) \geq lb$ for $b \leq u \leq \frac{b}{\gamma}$;

(ii) $f(u) \leq m'c$ for $0 \leq u \leq c$.

Then (4), (5) has at least one positive solution, where γ is given as in Theorem 2 and

$$l = \frac{1-\alpha\eta}{\gamma\eta\int_\eta^1(1-s)a(s)\mathrm{d}s}, \quad m' = \frac{1-\alpha\eta}{\int_0^1(1-s)a(s)\mathrm{d}s}.$$

Theorem 6[8] Assume (H_1), (H_2) hold and there are a, $b > 0$ such that

$$\varphi(a) = \max\big\{f(t,u) : 0 \leq t \leq 1,\, 0 \leq u \leq a\big\} \leq aA,$$

$$\psi(b) = \min\big\{f(t,u) : \mu \leq t \leq \nu,\, \sigma b \leq u \leq b\big\} \geq bB.$$

Then (3), (5) has at least one positive solution, where $\eta \leq \mu \leq \nu \leq 1$, $A = 2(1-\alpha\eta)$ and

$$B = \frac{2(1-\alpha\eta)}{\eta(\nu-\mu)(2-\nu-\mu)}, \quad \sigma = \min\Big\{\eta, \frac{1-\eta}{1-\alpha\eta}\Big\} \cdot \min_{t\in[\mu,\nu]}\Big\{\frac{t}{\eta}, \frac{(1-\alpha\eta)-(1-\alpha)t}{1-\eta}\Big\}.$$

Theorem 7 1) Assume (H_1)′, (H_2)′ hold and $f_\infty = \infty$. Then, for $\lambda > 0$ sufficiently small, (2), (6) has at least one positive solution;

2) Assume (H_1)′, (H_2)′ hold and there exist b, $c > 0$ such that $b < \rho c$ implies

(i) $f(u) \geq lb$ for $b \leq u \leq \frac{b}{\rho}$;

(ii) $f(u) \leq m'c$ for $0 \leq u \leq c$.

Then (4), (6) has at least one positive solution, where

$$\rho = \frac{\alpha(1-\eta)}{1-\alpha\eta}, \quad m' = \frac{1-\alpha}{\int_0^1(1-s)a(s)\mathrm{d}s}, \quad l = \frac{1-\alpha}{\rho(1-\eta)\int_0^\eta a(s)\mathrm{d}s + \rho\int_\eta^1(1-s)a(s)\mathrm{d}s}.$$

From now on, we concentrate on the two-point, m-point BVPs.

Theorem 8 Assume (H_2)′ holds and $f_\infty = \infty$. Then, for $\lambda > 0$ sufficiently small

1) (2), (7) has at least one positive solution;

2) (2), (8) has at least one positive solution;

3) (2), (9) has at least one positive solution.

Proof It is easy to check that (2), (7) is equivalent to the following integral equation

$$u(t) = \lambda\int_0^1 k(t,s)a(s)f\big(u(s)\big)\mathrm{d}s =: Tu(t), \quad u(t) \in C[0,1],$$

$$k(t,s) = \begin{cases} t(1-s), & t \leq s, \\ s(1-t), & t > s. \end{cases}$$

It is also easy to show that $T : P \to P$ is completely continuous by the Arzela-Ascoli theorem. From $k(t,s) \leq s(1-s)$, it follows that, for any $u \in P$,

$$\|Tu\| \leq \lambda\int_0^1 s(1-s)a(s)f\big(u(s)\big)\mathrm{d}s. \tag{12}$$

On the other hand, for $\frac{1}{4} \leq t \leq \frac{3}{4}$, one also has

$$k(t,s) \geq \begin{cases} \frac{1}{4}(1-s), & t \leq s, \\ \frac{1}{4}s, & t > s. \end{cases}$$

Hence, $k(t,s) \geq \frac{1}{4}s(1-s)$ for $\frac{1}{4} \leq t \leq \frac{3}{4}$, $0 \leq s \leq 1$. Therefore

$$\min_{t\in[\frac{1}{4},\frac{3}{4}]} Tu(t) \geq \frac{1}{4}\lambda \int_0^1 s(1-s)a(s)f\big(u(s)\big)\mathrm{d}s.$$

It follows from (12) that

$$\min_{t\in[\frac{1}{4},\frac{3}{4}]} Tu(t) \geq \frac{1}{4}\|Tu\|, \quad \forall\, u \in P.$$

Now let $\lambda \in (0,\varepsilon)$ be fixed, where $\varepsilon \in (0,1)$ is any given constant. From $f_\infty = \infty$, there is $b > 0$ such that $f(u) \geq Nu$ for any $u \geq b$, where the constant $N > 0$ satisfies

$$\frac{1}{4}N\lambda \int_{\frac{1}{4}}^{\frac{3}{4}} k\Big(\frac{1}{2}, s\Big)a(s)\mathrm{d}s > 1.$$

Thus, for $b \leq u(t) \leq 4b$, $t \in [\frac{1}{4}, \frac{3}{4}]$, it follows that

$$\begin{aligned} Tu\Big(\frac{1}{2}\Big) &= \lambda \int_0^1 k\Big(\frac{1}{2}, s\Big)a(s)f\big(u(s)\big)\mathrm{d}s \\ &\geq \lambda \int_{\frac{1}{4}}^{\frac{3}{4}} k\Big(\frac{1}{2}, s\Big)a(s)f\big(u(s)\big)\mathrm{d}s \geq Nb\lambda \int_{\frac{1}{4}}^{\frac{3}{4}} k\Big(\frac{1}{2}, s\Big)a(s)\mathrm{d}s > 4b. \end{aligned}$$

Consequently

$$\min_{t\in[\frac{1}{4},\frac{3}{4}]} Tu(t) \geq \frac{1}{4}\|Tu\| \geq \frac{1}{4}Tu\Big(\frac{1}{2}\Big) > b.$$

Therefore, the condition (ii) of Theorem 1 is satisfied. If $q > 0$, then

$$\beta(q) = \sup_{u\in P,\|u\|\leq q} \max_{t\in[0,1]} \Big[\int_0^1 k(t,s)a(s)f\big(u(s)\big)\mathrm{d}s\Big] > 0.$$

For any $c > 4b$, let $\pi = \frac{c}{\beta(c)}$, then for $\lambda \in (0,\varepsilon) \cap (0,\pi]$, and $u \in \bar{P}_c$, we have

$$Tu(t) \leq \pi\Big[\int_0^1 k(t,s)a(s)f\big(u(s)\big)\mathrm{d}s\Big] \leq \pi\beta(c) = c.$$

Thus, $\|Tu\| \leq c$, $\forall\, u \in \bar{P}_c$. The conclusion 1) follows from Theorem 1. Next we consider (2), (8) and (2), (9). (2), (8) and (2), (9) are equivalent to the following integral equations

$$u(t) = \lambda \int_0^1 k_1(t,s)a(s)f\big(u(s)\big)\mathrm{d}s =: T_1u(t),$$

$$u(t) = \lambda \int_0^1 k_2(t,s)a(s)f\big(u(s)\big)\mathrm{d}s =: T_2u(t),$$

respectively, where $u(t) \in C[0,1]$ and

$$k_1(t,s) = \begin{cases} t, & t \leq s, \\ s, & t > s, \end{cases} \qquad k_2(t,s) = \begin{cases} 1-s, & t \leq s, \\ 1-t, & t > s. \end{cases}$$

We can easily prove that

$$\min_{t\in[\frac{1}{2},1]} T_1u(t) \geq \frac{1}{2}\|T_1u\|, \quad \min_{t\in[0,\frac{1}{2}]} T_2u(t) \geq \frac{1}{2}\|T_2u\|, \quad \forall\, u \in P.$$

By using the same method as above, both (2), (8) and (2), (9) have positive solutions.

Theorem 9 Assume $(H_2)'$ holds and there are b, $c > 0$ such that $c > 4b$ implies

(i) $f(u) \geq lb$ for $b \leq u \leq 4b$;

(ii) $f(u) \leq mc$ for $0 \leq u \leq c$.

Then (4), (7) has at least one positive solution, where

$$l = \frac{17}{\int_{\frac{1}{4}}^{\frac{3}{4}} s(1-s)a(s)\mathrm{d}s}, \quad m = \frac{1}{\int_0^1 (1-s)a(s)\mathrm{d}s}.$$

Remark 1 (i) For (4), (8) or (4), (9), we have the similar results as those in Theorem 9;

(ii) For (1), (7)-(9) and (3), (7)-(9), we also have some results similar to Theorem 2 and Theorem 5.

Now we improve some results from [11-14] and obtain similar results by using the same method. We only give the results and omit their proofs. We make the following assumptions:

(K) There exist constants b, c, m', $l > 0$, $\tau \in (0,1)$ such that $b < \tau c$ implies

(A_1) $f(u) \geq lb$ for $b \leq u \leq \frac{b}{\tau}$;

(A_2) $f(u) \leq m'c$ for $0 \leq u \leq c$;

$(A_1)'$ $f(t,u) \geq lb$ for $0 \leq t \leq 1$, $b \leq u \leq \frac{b}{\tau}$;

$(A_2)'$ $f(t,u) \leq m'c$ for $0 \leq t \leq 1$, $0 \leq u \leq c$.

Where τ(m' or l) is different under different BVPs.

Theorem 10 1) Assume (H_2), (H_3) hold and $\underline{f}_{\infty} = \infty$. Then, for $\lambda > 0$ sufficiently small, (1), (10) has at least one positive solution;

2) Assume $(H_2)'$, (H_3) hold and $f_\infty = \infty$. Then, for $\lambda > 0$ sufficiently small, (2), (10) has at least one positive solution;

3) Assume $(H_2)'$, (H_3) and (K)-(A_1), (A_2) hold. Then, (4), (10) has at least one positive solution;

4) Assume (H_2), (H_3) and (K)-$(A_1)'$, $(A_2)'$ hold. Then (3), (10) has at least one positive solution;

5) Assume (H_3), (H_4) and (K)-(A_1), (A_2) hold. Then (4), (10) has at least one positive solution.

Theorem 11 1) Assume (H_2), (H_5) hold and $\underline{f}'_{\infty} = \infty$. Then, for $\lambda > 0$ sufficiently small, (1), (11) has at least one positive solution;

2) Assume $(H_2)'$, (H_5) hold and $f_\infty = \infty$. Then, for $\lambda > 0$ sufficiently small, the problem (2), (11) has at least one positive solution;

3) Assume $(H_2)'$, (H_5) and (K)-(A_1), (A_2) hold. Then (4), (11) has at least one positive solution;

4) Assume (H_2), (H_5) and (K)-$(A_1)'$, $(A_2)'$ hold. Then (3), (11) has at least one positive solution.

Remark 2 In Theorems 5, 7-11, we do not need the assumptions $f_0 = 0$, $f_\infty = \infty$ or $\overline{f}_0 = 0$, $\underline{f}_\infty = \infty$ which are required in [9-14]. From above discussions, we see that, by using Theorem 1, our conditions are weaker than those of the previous known results.

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几类边值问题的正解

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摘　要: 本文利用存在性定理，考察了二阶常微分方程两点、三点以及m-点边值问题正解的存在性．在较弱的条件下，给出了几类边值问题至少有一个正解存在的充分性条件．所得结果改进和推广了文献中的相应结论．

关键词: 正解；存在性定理；边值问题